Chapter 1
Introduction to Cellular Automata and Conway's Game of Life

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Although cellular automata has origins dating from the 1950s, widespread popular interest was not created until John Conway's "Game of Life" cellular automaton was initially revealed to the public in a 1970 Scientific American article [2]. The single feature of his "game" that probably caused this intensive interest was undoubtedly the discovery of "oscillators" (periodic forms) and "gliders" (translating oscillators).

1.1 A Brief Background

Cellular Automata (CA) can be constructed in one, two, three or more dimensions and can best be explained by giving an example utilizing Conway's rule. Start with an infinite grid of squares. Each individual square has 8 touching neighbors; typically these neighbors are treated the same (a "Moore neighborhood"), whether they touch a candidate square on a side or at a corner. We now fill in some of the squares; we shall say that these squares are "alive". Discrete time units called generations evolve; at each generation we apply a "rule" to the current configuration in order to arrive at the configuration for the next generation; in our example we shall use the rule below.

- If a live cell is touching 2 or 3 live cells (called "neighbors"), then it remains alive next generation, otherwise it dies.
- If a non-living cell is touching exactly 3 live cells, it comes to life next generation.

Figure 1.1 depicts the evolution of a simple configuration of filled-in ("live") cells for the above rule.

There are many notations for describing CA rules; these can differ depending upon the type of CA. For CA of more than one dimension, and in our present discussion, we shall utilize the following notation, which is standard for describing CA in two dimensions with Moore neighborhoods.

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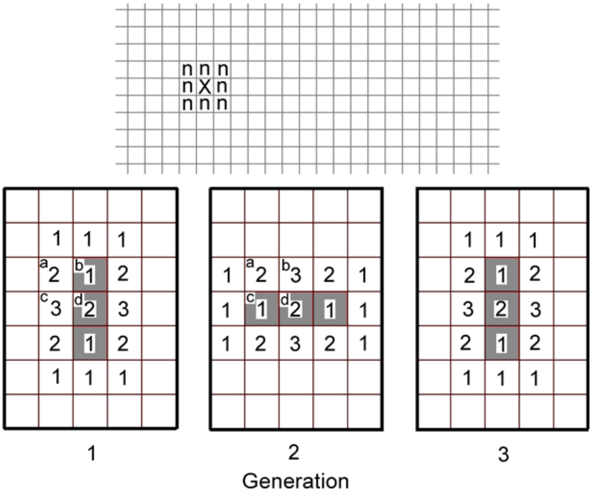

Fig. 1.1 *Top*: Each cell in a grid has 8 neighbors. The cells containing "*n*" are neighbors of the cell containing the "*X*". Any cell in the grid can be either "dead' or "alive". *Bottom*: Here we have outlined a specific area of what is presumably a much larger grid. At the *left* we have installed an initial shape. *Shaded cells* are alive; all others are dead. The number within each cell gives the quantity of live neighbors for that cell. (Cells containing no numbers have zero live neighbors.) Depicted are three generations, starting with the configuration at generation 1. Generations 2 then 3 show the result when we apply the following cellular automata rule: "Live cells with exactly 2 or 3 live neighbors remain alive (otherwise they die); dead cells with exactly 3 live neighbors come to life (otherwise they remain dead)". Let us now evaluate the transition from generation 1 to generation 2. In our diagram, cell "*a*" is dead. Since it does not have exactly 3 live neighbors, it remains dead. Cell "*b*" is alive, but it needs exactly 2 or 3 live neighbors to remain alive; since it only has 1, it dies. Cell "*c*" is dead; since it has exactly 3 live neighbors, it comes to life. And cell "*d*" has 2 live neighbors; hence it will remain alive. And so on. Notice that the form repeats every two generations. Such forms are called oscillators

We write a rule as

$$E_1, E_2, \ldots /F_1, F_2, \ldots$$

where the E_i ("environment") specify the number of live neighbors required to keep a living cell alive, and the F_i ("fertility") give the number required to bring a non-living cell to life. The E_i and F_i will be listed in ascending order; hence if $i > j$ then $E_i > E_j$, etc.

Thus the rule for the CA given above is 2,3/3. This rule, discovered by John Horton Conway, was examined in several articles in Scientific American and elsewhere, beginning with the seminal article in 1970 [2]. It is popularly known as Conway's "game of life". Of course it is not really a "game" in the usual sense, as the outcome is determined as soon as we pick a starting configuration.

Note that the shape in Fig. 1.1 repeats, with a period of two. A repeating form such as this is called an oscillator. Stationary forms can be considered oscillators with a period of one. In Figs. 1.2 and 1.3 we show several oscillators that

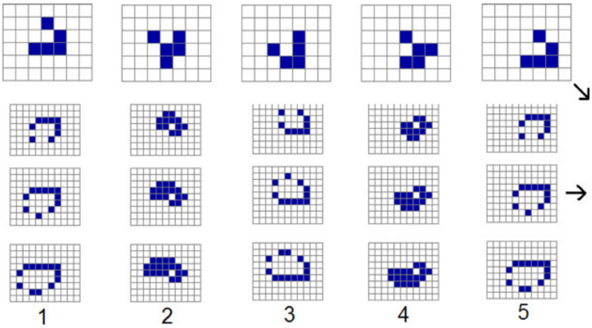

Fig. 1.2 Here we see a few of the small gliders that exist for 2,3/2. The form at the *top* — the original "glider" — was discovered by John Conway in 1968. The remaining forms were found shortly thereafter. Soon after Conway discovered rule 2,3/2 he started to give his various shapes rather whimsical names. That practice continues to this day. Hence, the name "glider" was given only to the simple shape at the *top*; the other gliders illustrated were called (from *top* to *bottom*) "lightweight spaceship", "middleweight spaceship" and "heavyweight spaceship". The numbers give the generation; each of the gliders shown has a period of four. The exact movement of each is depicted by its shifting position in the various small enclosing grids

move across the grid as they change from generation to generation. Such forms are called translating oscillators, or more commonly, gliders. Conway's rule popularized the term; in fact a flurry of activity began during which a great many shapes were discovered and exploited. These shapes were named whimsically — "blinker" (Fig. 1.1), "boat", "beehive" and an unbelievable myriad of others. Most translating oscillators were given names other than the simple moniker "glider" — there were "lightweight spaceships", "puffer trains", etc.

Of course rule 2,3/3 is not the only CA rule (even though it is the most interesting). Configurations under some rules always die out, and other rules lead to explosive growth. (We say that rules with expansive growth are unstable.)

We can easily find gliders for many unstable rules; for example Fig. 1.4 illustrates some simple constructs for rule 2/2. Note that it is practically impossible NOT to create gliders with this rule!

Hence we shall only look at gliders for rules that stabilize (i.e. exhibit bounded growth) and eventually yield only zero or more oscillators. We call such rules GoL (game of life) rules. "Stability" can be a rather murky concept, since there may be some carefully constructed forms within a GoL rule that grow without bounds.

Typically, such forms would never appear in random configurations. Hence, we shall informally define a GoL rule as follows:

1. All neighbors must be touching the candidate cell and all are treated the same (a Moore neighborhood).

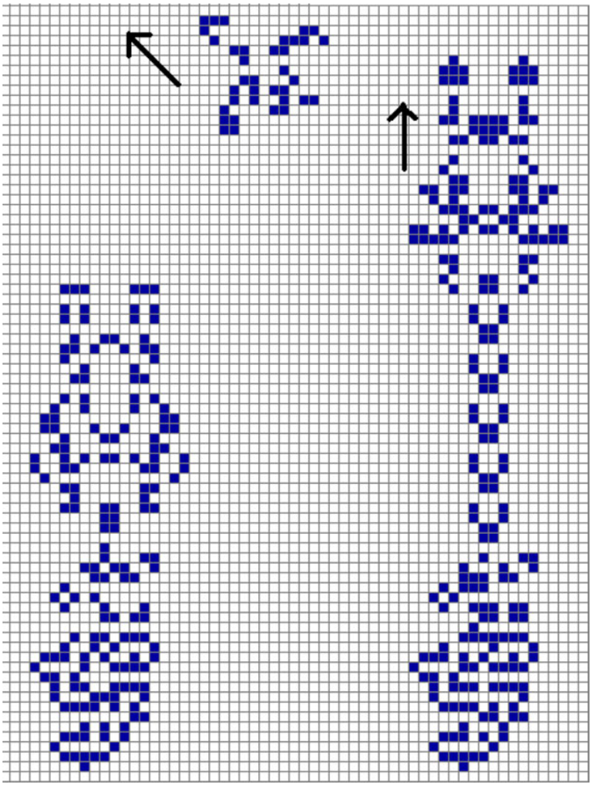

Fig. 1.3 Conway's rule 2,3/2 is rich with interesting forms — stationary or translating (i.e. gliders), as well as weird constructs that can expand forever. Here are but two of many hundreds that have been discovered. The glider at the *top* has a period of 5. The large form at the *left* is called a "wickstretcher" and grows forever. It is depicted at the *right* after 83 generations. The portion at the *top* of the wickstretcher moves up, while the blob at the *bottom* remains in place (though it does exhibit some turbulence). The "wick" in the *middle* increases in length and undulates downward like movie marquee lights

2. There must exist at least one translating oscillator (a glider).
3. Random configurations must eventually stabilize.

The above definition is a bit simplistic; for a more formal definition of a GoL rule refer to [1].

1.2 The Original Glider Gun

Conway's rule 2,3/3 is the original GoL rule and is unquestionably the most famous CA rule known. A challenge put forth by Conway was to create a configuration that would generate an ever increasing quantity of live cells. This challenge was met by William Gosper in 1970 — back when computing time was expensive and computers were slow by today's standards. He devised a form that spit out a continuous stream of gliders — a "glider gun" so to speak.

Interestingly, his gun configuration was displayed not as nice little squares, but as a rather primitive typewritten output (Fig. 1.5); this emphasizes the limited resources available in 1970 for seeking out such complex structures. Soon a "cottage industry" developed — all kinds of intricate initial configurations were discovered and exploited. Such research continues to this day and has pretty much

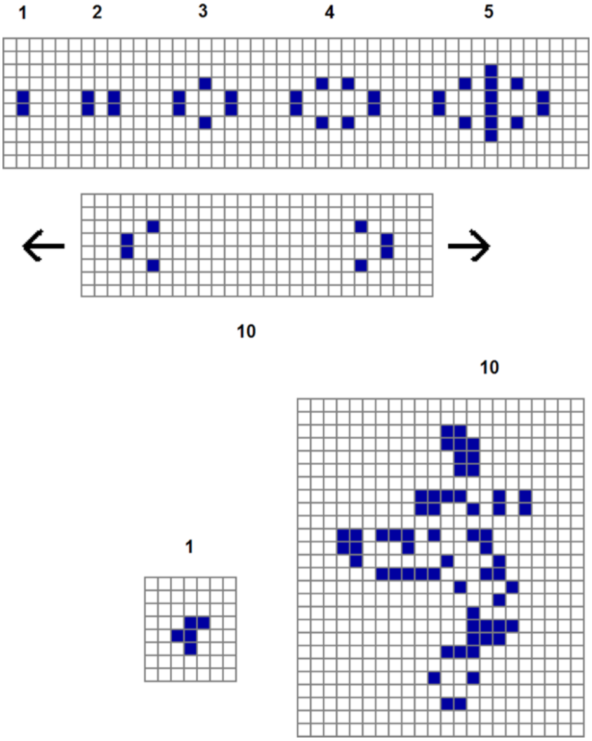

Fig. 1.4 Gliders exist under a large number of rules, but almost all such rules are unstable. For example the rule 2/2 exhibits rapid unbounded growth, and almost any starting configuration will yield "gliders"; e.g. just two live cells will produce two gliders going off in opposite directions. But almost any small form will quickly grow without bounds. The form at the *bottom left* expands to the shape at the *right* after only 10 generations. The generation is given with each form

guaranteed that Conway's Game of Life will always be the most famous CA rule.

1.3 Other GoL Rules in the Square Grid

The rule 2,4,5/3 is also a GoL rule and sports the glider shown in Fig. 1.6. It has not been seriously investigated and will probably not reveal the vast array of interesting forms that exist under 2,3/3. Interestingly, 2,3/3,8 appears to be a GoL rule which not surprisingly supports many of the constructs of 2,3/3. This ability to add terms of high neighbor counts onto known GoL rules, obtaining other GoL rules, seems to be easy to implement — particularly in higher dimensions or in grids with large neighbor counts such as the triangular grid, which has a neighbor count of 12.

1.4 Why Treat All Neighbors the Same?

By allowing only Moore neighborhoods in two (and higher) dimensions we greatly restrict the number of rules that can be written. And certainly we could consider

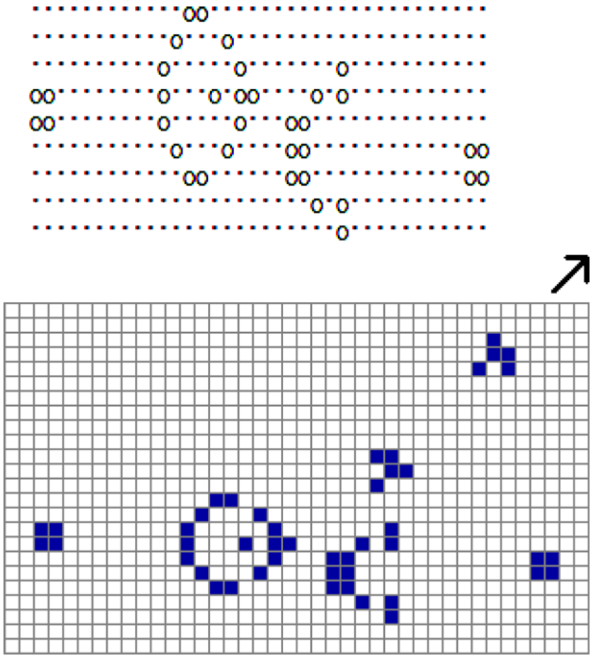

Fig. 1.5 A fascinating challenge was proposed by Conway in 1970 — he offered $50 to the first person who could devise a form for 2,3/2 that would generate an infinite number of living cells. One such form could be a "glider gun" — a construct that would create an endless stream of gliders. The challenge was soon met by William Gosper, then a student at MIT. His glider gun is illustrated here. At the *top*, testifying to the primitive computational power of the time, is an early illustration of Gosper's gun. At the *bottom* we see the gun in action, sending out a new glider every thirty generations (here it has sent out two gliders). Since 1970 there have been numerous such "guns" that generate all kinds of forms — some gliders and some stationary oscillators. Naturally in the latter case the generator must translate across the grid, leaving its intended stationary debris behind

"specialized" neighborhoods — e.g. treat as neighbors only those cells that touch on sides, or touch only the left two corners and nowhere else, or touch anywhere, but state in our rule that two or more live neighbors of a subject cell must not touch each other, etc. Consider the following rule for finding the next generation.

1. A living cell dies.
2. A dead cell comes to life if and only if its left side touches a live cell.

If we start, say, with a single cell we will obtain a "glider" of one cell that moves to the right one cell each generation! Such rules are easy to construct, as are more complex glider-producing positional rules. So we shall not investigate them further.

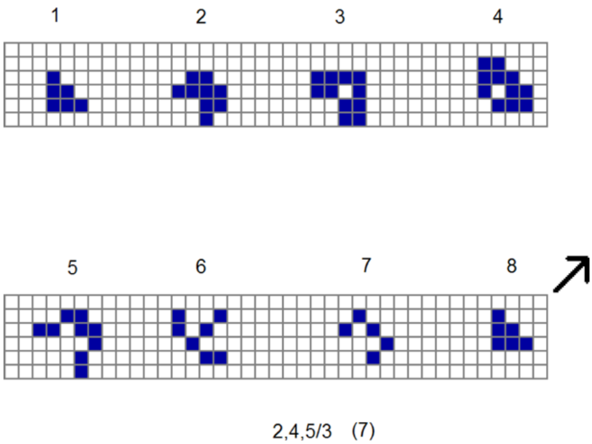

Fig. 1.6 There are a large number of interesting rules that can be written for the square grid but Rule 2,3/2 is undoubtedly the most fascinating — however it is not the only GoL rule. Here we depict a glider that has been found for the rule 2,4,5/3. And since that rule stabilizes, it is a valid GoL rule. Unfortunately it is not as interesting as 2,3/2 because its glider is not as likely to appear in random (and other) configurations — hence limiting the ability of 2,4,5/3 to produce interesting moving configurations. Note that the period is 7, indicated in *parentheses*

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